Simplify and then solve:

$(6^3)^2$

What is $6^3 \times 6^3$? ______

What is another way to write $(6^3)^2$?

______

$(3^4)^3$

What is 3^4 equal to? ______

______ × ______ × ______ = 531,441

What is another way to write $(3^4)^3$?

$(1^{12})^6$

What is $(1^{12})^6$ equal to? ______

What is another way to write $(1^{12})^6$?

Solve

$(-2^2)^3$

$(-4^3)^3$

$(-6^2)^4$

$(-2^3)^5$

$(-3^3)^2$

$(-4^1)^7$

$(-5^2)^2$

$(-2^3)^3$

The big question:

Do you see a pattern that results from the exponents?

What is the pattern

[redacted]

Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

Evaluate the Exponents

1) $(-10)^{2}$ = ______

2) $(8)^{2}$ = ______

3) $(3)^{5}$ = ______

4) $(7)^{3}$ = ______

5) $(-3)^{3}$ = ______

6) $(10)^{2}$ = ______

7) $(-4)^{3}$ = ______

8) $(-2)^{5}$ = ______

9) $(9)^{3}$ = ______

10) $(-5)^{2}$ = ______

11) $(-6)^{3}$ = ______

12) $(-7)^{3}$ = ______

13) $(-3)^{3}$ = ______

14) $(2)^{6}$ = ______

15) $(-9)^{3}$ = ______

16) $(-2)^{6}$ = ______

17) $(-8)^{3}$ = ______

18) $(3)^{4}$ = ______

19) $(12)^{2}$ = ______

20) $(4)^{3}$ = ______

0.3

Pre-Algebra - Order of Operations

Objective: Evaluate expressions using the order of operations, including the use of absolute value.

When simplifying expressions it is important that we simplify them in the correct order. Consider the following problem done two different ways:

Example 24.

$$\begin{array}{cl} \underbrace{2+5}\cdot 3 & \text{Add First} \\ \underbrace{7\cdot 3} & \text{Multiply} \\ 21 & \text{Solution} \end{array} \qquad \begin{array}{cl} 2+\underbrace{5\cdot 3} & \text{Multiply} \\ \underbrace{2+15} & \text{Add} \\ 17 & \text{Solution} \end{array}$$

The previous example illustrates that if the same problem is done two different ways we will arrive at two different solutions. However, only one method can be correct. It turns out the second method, 17, is the correct method. The order of operations ends with the most basic of operations, addition (or subtraction). Before addition is completed we must do repeated addition or multiplication (or division). Before multiplication is completed we must do repeated multiplication or exponents. When we want to do something out of order and make it come first we will put it in parenthesis (or grouping symbols). This list then is our order of operations we will use to simplify expressions.

Order of Operations:

Parenthesis (Grouping)

Exponents

Multiply and Divide (Left to Right)

Add and Subtract (Left to Right)

Multiply and Divide are on the same level because they are the same operation (division is just multiplying by the reciprocal). This means they must be done left to right, so some problems we will divide first, others we will multiply first. The same is true for adding and subtracting (subtracting is just adding the opposite).

Often students use the word PEMDAS to remember the order of operations, as the first letter of each operation creates the word PEMDAS. However, it is the author's suggestion to think about PEMDAS as a vertical word written as:

P
E
MD
AS

so we don't forget that multiplication and division are done left to right (same with addition and subtraction). Another way students remember the order of operations is to think of a phrase such as "Please Excuse My Dear Aunt Sally" where each word starts with the same letters as the order of operations start with.

World View Note: The first use of grouping symbols are found in 1646 in the Dutch mathematician, Franciscus van Schooten's text, Vieta. He used a bar over the expression that is to be evaluated first. So problems like $2(3+5)$ were written as $2\cdot\overline{3+5}$.

Example 25.

$$\begin{array}{rl}
2+3(\underbrace{9-4})^2 & \text{Parenthesis first}\\
2+3\underbrace{(5)^2} & \text{Exponents}\\
2+\underbrace{3(25)} & \text{Multiply}\\
\underbrace{2+75} & \text{Add}\\
77 & \text{Our Solution}
\end{array}$$

It is very important to remember to multiply and divide from from left to right!

Example 26.

$$\begin{array}{rl}
\underbrace{30\div 3}\cdot 2 & \text{Divide first (left to right!)}\\
\underbrace{10\cdot 2} & \text{Multiply}\\
20 & \text{Our Solution}
\end{array}$$

In the previous example, if we had multiplied first, five would have been the answer which is incorrect.

If there are several parenthesis in a problem we will start with the inner most parenthesis and work our way out. Inside each parenthesis we simplify using the order of operations as well. To make it easier to know which parenthesis goes with which parenthesis, different types of parenthesis will be used such as { } and [ ] and ( ), these parenthesis all mean the same thing, they are parenthesis and must be evaluated first.

Example 27.

$$\begin{array}{rl}
2\{8^2-7[32-4(\underbrace{3^2}+1)](-1)\} & \text{Inner most parenthesis, exponents first}\\
2\{8^2-7[32-4(\underbrace{9+1})](-1)\} & \text{Add inside those parenthesis}\\
2\{8^2-7[32-\underbrace{4(10)}](-1)\} & \text{Multiply inside inner most parenthesis}\\
2\{8^2-7[\underbrace{32-40}](-1)\} & \text{Subtract inside those parenthesis}\\
2\{\underbrace{8^2}-7[-8](-1)\} & \text{Exponents next}\\
2\{64-\underbrace{7[-8]}(-1)\} & \text{Multiply left to right, sign with the number}\\
2\{64+\underbrace{56(-1)}\} & \text{Finish multiplying}\\
2\{\underbrace{64-56}\} & \text{Subtract inside parenthesis}\\
\underbrace{2\{8\}} & \text{Multiply}\\
16 & \text{Our Solution}
\end{array}$$

As the above example illustrates, it can take several steps to complete a problem. The key to successfully solve order of operations problems is to take the time to show your work and do one step at a time. This will reduce the chance of making a mistake along the way.

There are several types of grouping symbols that can be used besides parenthesis. One type is a fraction bar. If we have a fraction, the entire numerator and the entire denominator must be evaluated before we reduce the fraction. In these cases we can simplify in both the numerator and denominator at the same time.

Example 28.

$$\frac{2^4-(-8)\cdot 3}{15\div 5-1} \quad \text{Exponent in the numerator, divide in denominator}$$

$$\frac{16-(-8)\cdot 3}{3-1} \quad \text{Multiply in the numerator, subtract in denominator}$$

$$\frac{16-(-24)}{2} \quad \text{Add the opposite to simplify numerator, denominator is done.}$$

$$\frac{40}{2} \quad \text{Reduce, divide}$$

$$20 \quad \text{Our Solution}$$

Another type of grouping symbol that also has an operation with it, absolute value. When we have absolute value we will evaluate everything inside the absolute value, just as if it were a normal parenthesis. Then once the inside is completed we will take the absolute value, or distance from zero, to make the number positive.

Example 29.

$$\begin{array}{rl}
1+3|-4^2-(-8)|+2|3+(-5)^2| & \text{Evaluate absolute values first, exponents} \\
1+3|-16-(-8)|+2|3+25| & \text{Add inside absolute values} \\
1+3|-8|+2|28| & \text{Evaluate absolute values} \\
1+3(8)+2(28) & \text{Multiply left to right} \\
1+24+2(28) & \text{Finish multiplying} \\
1+24+56 & \text{Add left to right} \\
25+56 & \text{Add} \\
81 & \text{Our Solution}
\end{array}$$

The above example also illustrates an important point about exponents. Exponents only are considered to be on the number they are attached to. This means when we see -4^2, only the 4 is squared, giving us $-(4^2)$ or -16. But when the negative is in parentheses, such as $(-5)^2$ the negative is part of the number and is also squared giving us a positive solution, 25.

0.3 Practice - Order of Operation

Solve.

1) $-6\cdot 4(-1)$

2) $(-6\div 6)^3$

3) $3+(8)\div|4|$

4) $5(-5+6)\cdot 6^2$

5) $8\div 4\cdot 2$

6) $7-5+6$

7) $[-9-(2-5)]\div(-6)$

8) $(-2\cdot 2^3\cdot 2)\div(-4)$

9) $-6+(-3-3)^2\div|3|$

10) $(-7-5)\div[-2-2-(-6)]$

11) $4-2|3^2-16|$

12) $\frac{-10-6}{(-2)^2}-5$

13) $[-1-(-5)]|3+2|$

14) $-3-\{3-[-3(2+4)-(-2)]\}$

15) $\frac{2+4|7+2^2|}{4\cdot 2+5\cdot 3}$

16) $-4-[2+4(-6)-4-|2^2-5\cdot 2|]$

17) $[6\cdot 2+2-(-6)](-5+\left|\frac{-18}{6}\right|)$

18) $2\cdot(-3)+3-6[-2-(-1-3)]$

19) $\frac{-13-2}{2-(-1)^3+(-6)-[-1-(-3)]}$

20) $\frac{-5^2+(-5)^2}{|4^2-2^5|-2\cdot 3}$

21) $6\cdot\frac{-8-4+(-4)-[-4-(-3)]}{(4^2+3^2)\div 5}$

22) $\frac{-9\cdot 2-(3-6)}{1-(-2+1)-(-3)}$

23) $\frac{2^3+4}{-18-6+(-4)-[-5(-1)(-5)]}$

24) $\frac{13+(-3)^2+4(-3)+1-[-10-(-6)]}{\{[4+5]\div[4^2-3^2(4-3)-8]\}+12}$

25) $\frac{5+3^2-24\div 6\cdot 2}{[5+3(2^2-5)]+|2^2-5|^2}$

Pre-Algebra - Integers

Objective: Add, Subtract, Multiply and Divide Positive and Negative Numbers.

The ability to work comfortably with negative numbers is essential to success in algebra. For this reason we will do a quick review of adding, subtracting, multiplying and dividing of integers. **Integers** are all the positive whole numbers, zero, and their opposites (negatives). As this is intended to be a review of integers, the descriptions and examples will not be as detailed as a normal lesson.

World View Note: The first set of rules for working with negative numbers was written out by the Indian mathematician Brahmagupta.

When adding integers we have two cases to consider. The first is if the signs match, both positive or both negative. If the signs match we will add the numbers together and keep the sign. This is illustrated in the following examples

Example 1.

$$\begin{array}{rl} -5+(-3) & \text{Same sign, add } 5+3, \text{ keep the negative} \\ -8 & \text{Our Solution} \end{array}$$

Example 2.

$$\begin{array}{rl} -7+(-5) & \text{Same sign, add } 7+5, \text{ keep the negative} \\ -12 & \text{Our Solution} \end{array}$$

If the signs don't match, one positive and one negative number, we will subtract the numbers (as if they were all positive) and then use the sign from the larger number. This means if the larger number is positive, the answer is positive. If the larger number is negative, the answer is negative. This is shown in the following examples.

Example 3.

$$\begin{array}{rl} -7+2 & \text{Different signs, subtract } 7-2, \text{ use sign from bigger number, negative} \\ -5 & \text{Our Solution} \end{array}$$

Example 4.

$$\begin{array}{rl} -4+6 & \text{Different signs, subtract } 6-4, \text{ use sign from bigger number, positive} \\ 2 & \text{Our Solution} \end{array}$$

Example 5.

$4+(-3)$	Different signs, subtract $4-3$, use sign from bigger number, positive
1	Our Solution

Example 6.

$7+(-10)$	Different signs, subtract $10-7$, use sign from bigger number, negative
-3	Our Solution

For subtraction of negatives we will change the problem to an addition problem which we can then solve using the above methods. The way we change a subtraction to an addition is to add the opposite of the number after the subtraction sign. Often this method is refered to as "add the opposite." This is illustrated in the following examples.

Example 7.

$8-3$	Add the opposite of 3
$8+(-3)$	Different signs, subtract $8-3$, use sign from bigger number, positive
5	Our Solution

Example 8.

$-4-6$	Add the opposite of 6
$-4+(-6)$	Same sign, add $4+6$, keep the negative
-10	Our Solution

Example 9.

$9-(-4)$	Add the opposite of -4
$9+4$	Same sign, add $9+4$, keep the positive
13	Our Solution

Example 10.

$-6-(-2)$	Add the opposite of -2
$-6+2$	Different sign, subtract $6-2$, use sign from bigger number, negative
-4	Our Solution

Multiplication and division of integers both work in a very similar pattern. The short description of the process is we multiply and divide like normal, if the signs match (both positive or both negative) the answer is positive. If the signs don't match (one positive and one negative) then the answer is negative. This is shown in the following examples

Example 11.

$$\begin{array}{rl} (4)(-6) & \text{Signs do not match, answer is negative} \\ -24 & \text{Our Solution} \end{array}$$

Example 12.

$$\begin{array}{rl} \dfrac{-36}{-9} & \text{Signs match, answer is positive} \\ 4 & \text{Our Solution} \end{array}$$

Example 13.

$$\begin{array}{rl} -2(-6) & \text{Signs match, answer is positive} \\ 12 & \text{Our Solution} \end{array}$$

Example 14.

$$\begin{array}{rl} \dfrac{15}{-3} & \text{Signs do not match, answer is negative} \\ -5 & \text{Our Solution} \end{array}$$

A few things to be careful of when working with integers. First be sure not to confuse a problem like $-3-8$ with $-3(-8)$. The second problem is a multiplication problem because there is nothing between the 3 and the parenthesis. If there is no operation written in between the parts, then we assume that means we are multiplying. The $-3-8$ problem, is subtraction because the subtraction separates the 3 from what comes after it. Another item to watch out for is to be careful not to mix up the pattern for adding and subtracting integers with the pattern for multiplying and dividing integers. They can look very similar, for example if the signs match on addition, the we keep the negative, $-3+(-7)=-10$, but if the signs match on multiplication, the answer is positive, $(-3)(-7)=21$.

0.1 Practice - Integers

Evaluate each expression.

1) $1-3$

2) $4-(-1)$

3) $(-6)-(-8)$

4) $(-6)+8$

5) $(-3)-3$

6) $(-8)-(-3)$

7) $3-(-5)$

8) $7-7$

9) $(-7)-(-5)$

10) $(-4)+(-1)$

11) $3-(-1)$

12) $(-1)+(-6)$

13) $6-3$

14) $(-8)+(-1)$

15) $(-5)+3$

16) $(-1)-8$

17) $2-3$

18) $5-7$

19) $(-8)-(-5)$

20) $(-5)+7$

21) $(-2)+(-5)$

22) $1+(-1)$

23) $5-(-6)$

24) $8-(-1)$

25) $(-6)+3$

26) $(-3)+(-1)$

27) $4-7$

28) $7-3$

29) $(-7)+7$

30) $(-3)+(-5)$

Find each product.

31) $(4)(-1)$

32) $(7)(-5)$

33) $(10)(-8)$

34) $(-7)(-2)$

35) $(-4)(-2)$

36) $(-6)(-1)$

37) $(-7)(8)$

38) $(6)(-1)$

39) $(9)(-4)$

40) $(-9)(-7)$

41) $(-5)(2)$

42) $(-2)(-2)$

43) $(-5)(4)$

44) $(-3)(-9)$