Warm Up
(Transparency)

Introductory Problem: Given 5 and 4, use one mathematical operation to get the highest number.

Discussion Questions.

1. What is the meaning of an exponent?

2. Does $5^4 = 4^5$? If they are not, which is greater?

3. Would 3^6 be greater than, less than, or equal to 6^3?

4. Given two different whole numbers would you get a greater answer, if you use the smaller number or the larger number in the base? Are there any exceptions? (Hint: Use questions 2 and 3.)

Name ______________________

Practice 2-4

Exponents

Write using exponents.

1. $3 \times 3 \times 3 \times 3$ ______

2. 364×364 ______

3. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ ______

4. $13 \times 13 \times 13$ ______

5. $8 \times 8 \times 8 \times 7 \times 7$ ______

6. 49 ______

Write in expanded form.

7. 10^4 ______

8. 6^5 ______

9. 3^2 ______

10. 7^3 ______

11. 12^4 ______

12. 5 cubed ______

Write in standard form.

13. 5^4 ______

14. 2^6 ______

15. 11 squared ______

16. 10^7 ______

17. 12^2 ______

18. 6 cubed ______

Compare using $<$, $>$, or $=$.

19. $4^2 \bigcirc 2^4$

20. $4^3 \bigcirc 3^4$

21. $5^8 \bigcirc 5^9$

22. $3^8 \bigcirc 3 \times 8$

23. $2^5 \bigcirc 5^2$

24. $10^3 \bigcirc 10 + 10 + 10$

25. $5^3 \bigcirc 5 \times 5 \times 5$

26. $7^3 \bigcirc 3^7$

27. $10^4 \bigcirc 4 \times 10$

For each number in exponential notation, identify the base, exponent, and power. Use a calculator to write each number in standard form.

28. A typical American kid watches about 18^4 television advertisements between birth and high school graduation.

base ______ exponent ______

power ______ standard form ______

29. The highest point in Kentucky is Black Mountain. Its height is about 2^{12} feet.

base ______ exponent ______

power ______ standard form ______

Introduction:
(For use on overhead)

Given a sheet of paper, how many times can I fold it? Would you say 6 or 7 times! Notice the initial rectangular shape decreases as the number of folds increases. Therefore, the number of rectangles increases as the number of folds increases.
How many rectangles are created?

The first fold of the paper creates two rectangles. The second fold creates four. If the paper is folded a third time, eight rectangles are formed.

Question: How many rectangles are formed on the fourth fold?

Answer: Sixteen. Notice that for each fold the number of rectangles double from the previous number.

If a table is created, the following pattern develops.

Folds	0	1	2	3	4	5	6	7	8	9	10
Rectangles	1	2	4	8	16	32	64	128	256	512	1024

Could the data be written in a different form? What about exponents?

If the form a^x is used, a represents the base and x the exponent. Then, in this situation, two would be our base and the number of folds represented by x. The new table of values would be

Folds	0	1	2	3	4	5	6	7	8	9	10
Rectangles	2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}

Question: How many rectangles are created after 30 folds?

Answer: 2^{30} or 1,073,741,824 rectangles.

This could be written as a function: $Y = 2^x$

Using a calculator, it is easy to calculate the number of rectangles as a number is substituted for x.

A TI-83 (Plus) can be used to show how this function is represented graphically.

Date: ________

Names: ______________________________________

Group Activity Sheet

You are at an interview for a job. While there, you are asked which salary you would prefer. Salary A would pay $500 the first day and give you a $100 daily increase. Salary B would pay 1 cent for the first day and every successive day you will be paid double the amount.

Questions:

1. Create a table of values that would show patterns for salary A and salary B.

# of Days	1	2	3	4	5	10	15	....	19
Pay A									
Pay B									

2. Determine the function for each salary.
   Payment A:

   Payment B:

3. By using the equations, sketch the graphs of the two functions. Tell where the lines intersect and explain what it means.

4 Would you rather be given salary A or salary B? Explain.

Chapter 5 : Polynomials

5.1

Polynomials - Exponent Properties

Objective: Simplify expressions using the properties of exponents.

Problems with expoenents can often be simplified using a few basic exponent properties. Exponents represent repeated multiplication. We will use this fact to discover the important properties.

World View Note: The word exponent comes from the Latin "expo" meaning out of and "ponere" meaning place. While there is some debate, it seems that the Babylonians living in Iraq were the first to do work with exponents (dating back to the 23rd century BC or earlier!)

Example 196.

$$\begin{array}{rl} a^3a^2 & \text{Expand exponents to multiplication problem} \\ (a\,a\,a)(a\,a) & \text{Now we have } 5\,a's \text{ being multiplied together} \\ a^5 & \text{Our Solution} \end{array}$$

A quicker method to arrive at our answer would have been to just add the exponents: $a^3a^2 = a^{3+2} = a^5$ This is known as the **product rule of exponents**

Product Rule of Exponents: $a^m a^n = a^{m+n}$

The product rule of exponents can be used to simplify many problems. We will add the exponent on like variables. This is shown in the following examples

Example 197.

$$\begin{array}{rl} 3^2\cdot 3^6\cdot 3 & \text{Same base, add the exponents } 2+6+1 \\ 3^9 & \text{Our Solution} \end{array}$$

Example 198.

$$\begin{array}{rl} 2x^3y^5z\cdot 5xy^2z^3 & \text{Multiply } 2\cdot 5, \text{ add exponents on } x, y \text{ and } z \\ 10x^4y^7z^4 & \text{Our Solution} \end{array}$$

Rather than multiplying, we will now try to divide with exponents

Example 199.

$$\begin{array}{rl} \dfrac{a^5}{a^2} & \text{Expand exponents} \\ \dfrac{a\,a\,a\,a\,a}{a\,a} & \text{Divide out two of the } a's \\ a\,a\,a & \text{Convert to exponents} \\ a^3 & \text{Our Solution} \end{array}$$

A quicker method to arrive at the solution would have been to just subtract the exponents, $\frac{a^5}{a^2} = a^{5-2} = a^3$. This is known as the quotient rule of exponents.

$$\textbf{Quotient Rule of Exponents: } \frac{a^m}{a^n} = a^{m-n}$$

The quotient rule of exponents can similarly be used to simplify exponent problems by subtracting exponents on like variables. This is shown in the following examples.

Example 200.

$$\begin{array}{rl} \dfrac{7^{13}}{7^5} & \text{Same base, subtract the exponents} \\ 7^8 & \text{Our Solution} \end{array}$$

Example 201.

$$\begin{array}{rl} \dfrac{5a^3b^5c^2}{2\mathrm{ab}^3c} & \text{Subtract exponents on } a, b \text{ and } c \\ \\ \dfrac{5}{2}a^2b^2c & \text{Our Solution} \end{array}$$

A third property we will look at will have an exponent problem raised to a second exponent. This is investigated in the following example.

Example 202.

$$\begin{array}{rl} \left(a^2\right)^3 & \text{This means we have } a^2 \text{ three times} \\ a^2 \cdot a^2 \cdot a^2 & \text{Add exponents} \\ a^6 & \text{Our solution} \end{array}$$

A quicker method to arrive at the solution would have been to just multiply the exponents, $(a^2)^3 = a^{2\cdot 3} = a^6$. This is known as the power of a power rule of exponents.

$$\textbf{Power of a Power Rule of Exponents: } (a^m)^n = a^{mn}$$

This property is often combined with two other properties which we will investigate now.

Example 203.

$$\begin{array}{rl} (ab)^3 & \text{This means we have } (ab) \text{ three times} \\ (ab)(ab)(ab) & \text{Three } a's \text{ and three } b's \text{ can be written with exponents} \\ a^3b^3 & \text{Our Solution} \end{array}$$

A quicker method to arrive at the solution would have been to take the exponent of three and put it on each factor in parenthesis, $(ab)^3 = a^3b^3$. This is known as the power of a product rule or exponents.

$$\textbf{Power of } a \textbf{ Product Rule of Exponents: } (\mathbf{ab})^m = a^m b^m$$

It is important to be careful to only use the power of a product rule with multiplication inside parenthesis. This property does NOT work if there is addition or subtraction.

Warning 204.

$$(a+b)^m \neq a^m + b^m \qquad \text{These are } \textbf{NOT} \text{ equal, beware of this error!}$$

Another property that is very similar to the power of a product rule is considered next.

Example 205.

$$\left(\frac{a}{b}\right)^3 \qquad \text{This means we have the fraction three timse}$$

$$\left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right) \qquad \text{Multiply fractions across the top and bottom, using exponents}$$

$$\frac{a^3}{b^3} \qquad \text{Our Solution}$$

A quicker method to arrive at the solution would have been to put the exponent on every factor in both the numerator and denominator, $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$. This is known as the power of a quotient rule of exponents.

$$\textbf{Power of } a \textbf{ Quotient Rule of Exponents: } \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

The power of a power, product and quotient rules are often used together to simplify expressions. This is shown in the following examples.

Example 206.

$$(x^3yz^2)^4 \qquad \text{Put the exponent of 4 on each factor, multiplying powers}$$

$$x^{12}y^4z^8 \qquad \text{Our solution}$$

Example 207.

$$\left(\frac{a^3b}{c^8d^5}\right)^2 \quad \text{Put the exponent of 2 on each factor, multiplying powers}$$

$$\frac{a^6b^2}{c^8d^{10}} \quad \text{Our Solution}$$

As we multiply exponents its important to remember these properties apply to exponents, not bases. An expressions such as 5^3 does not mean we multipy 5 by 3, rather we multiply 5 three times, $5 \times 5 \times 5 = 125$. This is shown in the next example.

Example 208.

$(4x^2y^5)^3$	Put the exponent of 3 on each factor, multiplying powers
$4^3x^6y^{15}$	Evaluate 4^3
$64x^6y^{15}$	Our Solution

In the previous example we did not put the 3 on the 4 and multipy to get 12, this would have been incorrect. Never multipy a base by the exponent. These properties pertain to exponents only, not bases.

In this lesson we have discussed 5 different exponent properties. These rules are summarized in the following table.

Rules of Exponents

Product Rule of Exponents	$a^ma^n = a^{m+n}$
Quotient Rule of Exponents	$\dfrac{a^m}{a^n} = a^{m-n}$
Power of a Power Rule of Exponents	$(a^m)^n = a^{mn}$
Power of a Product Rule of Exponents	$(ab)^m = a^mb^m$
Power of a Quotient Rule of Exponents	$\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

These five properties are often mixed up in the same problem. Often there is a bit of flexibility as to which property is used first. However, order of operations still applies to a problem. For this reason it is the suggestion of the auther to simplify inside any parenthesis first, then simplify any exponents (using power rules), and finally simplify any multiplication or division (using product and quotient rules). This is illustrated in the next few examples.

Example 209.

$(4x^3y \cdot 5x^4y^2)^3$	In parenthesis simplify using product rule, adding exponents
$(20x^7y^3)^3$	With power rules, put three on each factor, multiplying exponents
$20^3x^{21}y^9$	Evaluate 20^3
$8000x^{21}y^9$	Our Solution

Example 210.

$7a^3(2a^4)^3$ Parenthesis are already simplified. next use power rules

$7a^3(8a^{12})$ Using product rule, add exponents and multiply numbers

$56a^{15}$ Our Solution

Example 211.

$\dfrac{3a^3b\cdot 10a^4b^3}{2a^4b^2}$ Simplify numerator with product rule, adding exponents

$\dfrac{30a^7b^4}{2a^4b^2}$ Now use the quotient rule to subtract exponents

$15a^3b^2$ Our Solution

Example 212.

$\dfrac{3m^8n^{12}}{(m^2n^3)^3}$ Use power rule in denominator

$\dfrac{3m^8n^{12}}{m^6n^9}$ Use quotient rule

$3m^2n^3$ Our solution

Example 213.

$\left(\dfrac{3ab^2(2a^4b^2)^3}{6a^5b^7}\right)^2$ Simplify inside parenthesis first, using power rule in numerator

$\left(\dfrac{3ab^2(8a^{12}b^6)}{6a^5b^7}\right)^2$ Simplify numerator using product rule

$\left(\dfrac{24a^{13}b^8}{6a^5b^7}\right)^2$ Simplify using the quotient rule

$(4a^8b)^2$ Now that the parenthesis are simplified, use the power rules

$16a^{16}b^2$ Our Solution

Clearly these problems can quickly become quite involved. Remember to follow order of operations as a guide, simplify inside parenthesis first, then power rules, then product and quotient rules.

5.1 Practice - Exponent Properties

Simplify.

1) $4 \cdot 4^4 \cdot 4^4$

2) $4 \cdot 4^4 \cdot 4^2$

3) $4 \cdot 2^2$

4) $3 \cdot 3^3 \cdot 3^2$

5) $3m \cdot 4mn$

6) $3x \cdot 4x^2$

7) $2m^4n^2 \cdot 4nm^2$

8) $x^2y^4 \cdot xy^2$

9) $(3^3)^4$

10) $(4^3)^4$

11) $(4^4)^2$

12) $(3^2)^3$

13) $(2u^3v^2)^2$

14) $(xy)^3$

15) $(2a^4)^4$

16) $(2xy)^4$

17) $\frac{4^5}{4^3}$

18) $\frac{3^7}{3^3}$

19) $\frac{3^2}{3}$

20) $\frac{3^4}{3}$

21) $\frac{3nm^2}{3n}$

22) $\frac{x^2y^4}{4xy}$

23) $\frac{4x^3y^4}{3xy^3}$

24) $\frac{xy^3}{4xy}$

25) $(x^3y^4 \cdot 2x^2y^3)^2$

26) $(u^2v^2 \cdot 2u^4)^3$

27) $2x(x^4y^4)^4$

28) $\frac{3vu^5 \cdot 2v^3}{uv^2 \cdot 2u^3v}$

29) $\frac{2x^7y^5}{3x^3y \cdot 4x^2y^3}$

30) $\frac{2ba^7 \cdot 2b^4}{ba^2 \cdot 3a^3b^4}$

31) $\left(\frac{(2x)^3}{x^3}\right)^2$

32) $\frac{2a^2b^2a^7}{(ba^4)^2}$

33) $\left(\frac{2y^{17}}{(2x^2y^4)^4}\right)^3$

34) $\frac{yx^2 \cdot (y^4)^2}{2y^4}$

35) $\left(\frac{2mn^4 \cdot 2m^4n^4}{mn^4}\right)^3$

36) $\frac{n^3(n^4)^2}{2mn}$

37) $\frac{2xy^5 \cdot 2x^2y^3}{2xy^4 \cdot y^3}$

38) $\frac{(2y^3x^2)^2}{2x^2y^4 \cdot x^2}$

39) $\frac{q^3r^2 \cdot (2p^2q^2r^3)^2}{2p^3}$

40) $\frac{2x^4y^5 \cdot 2z^{10}x^2y^7}{(xy^2z^2)^4}$

41) $\left(\frac{zy^3 \cdot z^3x^4y^4}{x^3y^3z^3}\right)^4$

42) $\left(\frac{2q^3p^3r^4 \cdot 2p^3}{(qrp^3)^2}\right)^4$

43) $\frac{2x^2y^2z^6 \cdot 2zx^2y^2}{(x^2z^3)^2}$