A Quick Algebra Review

Simplifying Expressions

An **expression** is a mathematical "phrase." Expressions contain numbers and variables, but not an equal sign. An ***equa*tion** has an "equal" sign. For example:

Expression:	Equation:
$5 + 3$	$5 + 3 = 8$
$x + 3$	$x + 3 = 8$
$(x + 4)(x - 2)$	$(x + 4)(x - 2) = 10$
$x^2 + 5x + 6$	$x^2 + 5x + 6 = 0$
$x - 8$	$x - 8 > 3$

When we **simplify** an expression, we work until there are as few terms as possible. This process makes the expression easier to use, (that's why it's called "simplify"). The first thing we want to do when simplifying an expression is to **combine like terms**.

Let's try another one…

Inside the parenthesis, look for order of operation rules - PEMDAS.
We need to subtract 5 from 3 then add 12 inside the parentheses. This takes care of the P in PEMDAS, now for the E, Exponents. We square -4. Make sure to use $(-4)^2$ if you are relying on your calculator. If you input -4^2 the calculator will evaluate the expression using PEMDAS. It will do the exponent first, then multiply by -1, giving you **-16**, though we know the answer is **16**. Now we can multiply and then add to finish up.

Simplify:

$(-4)^2 + 2[12 + (3-5)]$

$= (-4)^2 + 2[12 + (-2)]$

$= (-4)^2 + 2[10]$

$= 16 + 2[10]$

$= 16 + 20$

$= 36$

Practice makes perfect…

Since there are no like terms inside the parenthesis, we need to distribute the negative sign and then see what we have. There is really a -1 there but we're basically lazy when it comes to the number one and don't always write it (since 1 times anything is itself). So we need to take -1 times **EVERYTHING** in the parenthesis, not just the first term. Once we have done that, we can combine like terms and rewrite the expression.

Simplify:

$(5a^2 - 3a + 1) - (2a^2 - 4a + 6)$

$= (5a^2 - 3a + 1) - 1(2a^2 - 4a + 6)$

$= (5a^2 - 3a + 1) - 1(2a^2) - (-1)(-4a) + (-1)(6)$

$= (5a^2 - 3a + 1) - 2a^2 + 4a - 6$

$= 5a^2 - 3a + 1 - 2a^2 + 4a - 6$

$= 3a^2 + a - 5$

Now you try: $2x + 4[2 - (5x - 3)]$

[you should get $-18x + 20$]

Problem Solving

Many people look at word problems and think, "I'm really bad at these!" But once we accept them, they help us solve problems in life when the equation, numbers, and variables are not given to us. They help us THINK, logically.

One of the challenging parts of solving word problems is that you to take a problem given in written English and translate it into a mathematical equation. In other words, we turn *words* into *numbers, variables, and mathematical symbols*.

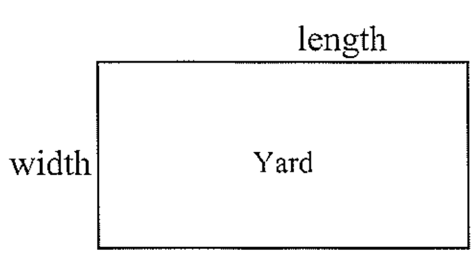
There are three important steps to "translating" a word problem into an equation we can work with:

1. Understand the problem
2. Define the variables
3. Write an equation

Let's look at an example:

> *The fence around my rectangular back yard is 48 feet long. My yard is 3ft longer than twice the width. What is the width of my yard? What is the length?*

First, we have to make sure we **understand the problem**. So what's going on here? Drawing a picture often helps with this step.



We know that the problem is describing a person's rectangular yard. We also know that one side is the width and the other side is the length. The **perimeter** of (distance around) the yard is 48ft. To arrive at that perimeter, we add length + length + width + width, or use the formula $2l + 2w = p$ (l = length, w = width, p = perimeter)

Check to see that it works

$2(7) + 3$
$14 + 3$

$17\text{ft} = \text{length}$

Substitute our newly found width and simplify using order of operations ☺ So now we know that the yard is 7ft wide and 17ft long.

Your turn:

I have a box. The length of the box is 12 in. The height of the box is 5 in. The box has a total volume of 360 in.^{2} What is the width of the box?
Note: *The formula for volume is $V = lwh$, where v = volume, l = length, w = width, and h = height.*

(You should get $w = 6$in.)

<u>Inequalities</u>

We've all been taught little tricks to remember the inequality sign. For example; when given $x < 10$, we know that x is **less than** ten because <u>x has the LITTLE side of the sign and 10 has the BIG side of the sign.</u>

Solving inequalities is similar to solving equations; what you do to one side of an inequality, we must do to the other. If we are given $\mathbf{x + 7 > 13}$ and asked to solve, we would undo the addition on the left side by subtracting 7 from both sides. We would then be left with $\mathbf{x > 6}$, which is our answer.

Suppose we were given $\mathbf{\frac{1}{4} x < 2}$. To undo the division, we would multiply both sides by 4. The result would be $\mathbf{x < 8}$.

But if we had $\mathbf{-\frac{1}{4} x < 2}$, we would multiply by both sided of the inequality by -4 and the rule is that **when multiplying (or dividing) by a *negative number*, we must always *flip the sign of an inequality*.** So we would get $\mathbf{x > 2}$.

$x + 2 = 3$ and $x + 2 = -3$

$-1 \quad -1 \qquad\qquad -2 \quad -2$

$x = 1 \qquad\qquad x = -5$

Your turn: $|3x - 4| = 5$

[You should get x = -1/3 and x = 3]

Linear Equations

If we plot all the solutions to a linear equation on a graph, they form a line. That is why they are called **linear equations**. Linear equations can be written in **slope-intercept form**. There are two important things to know when writing the equation of a line in **slope-intercept form**; the **slope** and the **y-intercept.**

The **slope** of the line gives the rate of change. Remember slope is rise over the run. That is because from any point on the line, the "direction" to another point on the line is given in terms of the x and y coordinates, how much it "rises," goes up (or if negative, down) and "runs" (moves in the horizontal direction). For example, if the slope is 2/3; from any point on your line to get to another point on your line we would move positive (up) 2 and positive (to the right) 3. The **y-intercept** is the point where the line crosses the y axis.

Slope-intercept form is written where the equation is solved for y:

$$\mathbf{y = mx + b}$$

where **m** is the slope and **b** is the y-intercept.

To find the slope of a line that is not written in slope intercept form, we must solve the equation for y.

For example:

Find the slope and y-intercept of the line $3x - 2y = 4$.

We need to put the equation in slope intercept form. Then we can quickly identify the parts by their position in the equation.

Now you have everything you need to write the equation of the line with slope -2/3 through the point (9,2). Put the slope and y-intercept back into the equation and you have y = -2/3x + 8.

Sometimes, the slope isn't given. Let's try another example:

Find the equation of a line that passes through the points (3, 1) and (6, 2).

The first thing we need to do is find the slope of the line through these two points.

Remember that:

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2-1}{6-3}$$

$$m = \frac{1}{3}$$

No one is trying to confuse you with all of these subscripts…Its just that we have two points (x, y) so we call the first one (x_1, y_1) and the second (x_2, y_2) Substitute (3, 1) in for one point and (6, 2) in for the other. Be careful to go the same direction when you subtract and WATCH YOUR SIGNS!

Now we can substitute the slope and one of the points (either one will work) into the equation y =mx + b and solve for b.

We'll use the point (3, 1) for (x, y)

y = mx + b
1 = 1/3 (3) + b
1 = 1 + b
1 - 1 = b
0 = b

Now we have everything we need to write the equation of the line with points (3, 1) and (6, 2). We found that the slope m =1/3 then we found the y-intercept. Putting them into the equation you have y = 1/3x + 0

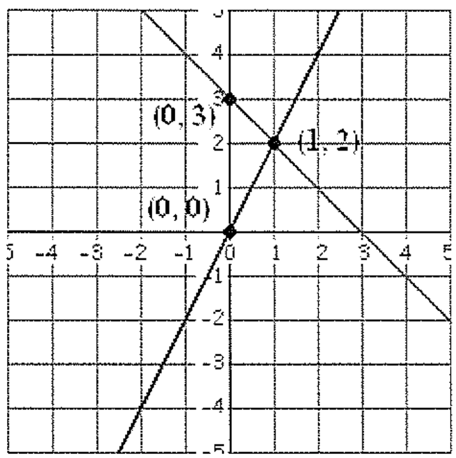
Your turn: Find the equation of the line that goes through the points (5, 4) and (1, -4).

[You should get y = 2x - 6]

is called a **system of equations**. The solution to a system of equations must satisfy *both* equations. There are several different methods to accomplish this. We will review two: **graphing** and **substitution**.

Graphing:

What does the graph for a system of equations look like? Well, one equation makes a line, so two equations make....two lines! For the above example, the graph would look like this:



$y = 2x$
$y = -x + 3$

Since each point on a line is a solution to the equation – the point that satisfies both equations is the point where they intersect. The solution for this system of equations is the point (1, 2).

Your turn: Solve the system of equations by graphing:

$$\mathbf{y = -1/2\ x - 2}$$
$$\mathbf{y = -7/2\ x + 4}$$

[You should get (2, -3)]

Note: Often we need to solve the equations for y-intercept form before graphing them.

Your turn: solve by substitution:

$$x = 3y$$
$$y = x + 4$$

[You should get (-6, -2)]

Laws of Exponents

The laws of exponents give us rules for dealing with powers of variables and numbers that have the **same base**. Here are some of the basics.

Law	Explanation	Example
$A^n = (A*A*A*A\ldots n$ times)	An exponent is a base (A) multiplied by itself n times.	$5^4 = 5*5*5*5 = 625$
$A^m*A^n = A^{m+n}$	When multiplying two powers, add the exponents.	$x^5*x^2 = x^7$
$(A^m)^n = A^{m*n}$	When raising a power to another power, multiply the exponents.	$(y^4)^2 =$
$\frac{A^n}{A^m} = A^{n-m}$	When dividing powers, subtract the power of the denominator(bottom) exponent from the numerator(upper) one.	$\frac{2^3}{2^2} = 2^{3-2} = 2^1 = 2$
$A^0 = 1$	Any number raised to the power of 0 equals 1	$587^0 = 1$
$A^{-n} = \frac{1}{A^n}$	When you have a negative exponent, it means **inverse**, (the negative exponent is an operation that "flips" only the base that it applies to).	$x^{-3} = \frac{1}{x^3}$ $\frac{5}{x^{-3}} = 5x^3$

$(2x+1)(x-2)$

$2x^2 - 2x$

$(2x+1)(x-2)$

$2x^2 - 2x + x - 2$

$2x^2 - x - 2$

Our first step is to multiply the *first* terms together (red). Then we multiply the *outer* terms (blue).

Next, we multiply the *inner* terms (green), followed by the *last* terms (pink).

Now, we combine like terms and our answer is simplified!

Your turn: $(2x + 3)(x - 1)$

[You should get $2x^2 + x - 3$]

Quadratics

Quadratic equations are equations that have a variable to the second power, like $x^2 + x = 6$. Since x^2 and x are not like terms they can not be combined. We need a new way for finding solutions to quadratic equations.

Solving by factoring:

To solve an equation by factoring, one side of the equation must be equal to zero. In the equation $3x^2 = x$. We would need to subtract **x** from both sides, so we would have $3x^2 - x = 0$.

Our goal with factoring is to find two terms that multiply together to give us zero. Since **x** is a factor in both $3x^2$ and **x**, we can **factor out** an x from the equation and rewrite it, $x(3x - 1) = 0$

+ 2, we get -1, but if we add (-2) + 3, we get 1, which is exactly what we are looking for. We now know that one set of parentheses must have a -2, and the other must have a three.

$$(x - 2)(x + 3) = 0$$

Again, we know that two things multiplied together will only equal zero if one of them is zero. So we set **x – 2** and **x + 3** equal to zero and solve for x.

$$\mathbf{x - 2 = 0} \qquad \mathbf{x + 3 = 0}$$
$$x = 2 \qquad x = -3$$

Our solutions to the quadratic equation are **x = -3 and 2**.

Let's try another example:

$\mathbf{2x^2 - 7x - 4 = 0}$

This one's a bit harder because of the 2 in front of the x^2. When setting up our parentheses, we have to think about how our **F**irst terms can multiply together to give us 2. The only way to do this is by 2x and x.

(2x)(x)

(2x + 1)(x – 4)

Here, it really becomes a puzzle. We have to find two numbers that **multiply** together to give us four. Because of the 2 in front of the x^2, we know that *double* one of these numbers plus the other must give us -7. Luckily 4 has only two sets of factors: 1 & 4 and 2 & 2. One of these must be negative and one positive, since the 4 is negative. If you double -4, you get -8. If you add 1 to -8, you get -7, which is exactly what we are looking for. Since the 4 *must* be multiplied by the 2 in the first set of parentheses, it has to go in the other set of parenthesis.

$$\mathbf{2x + 1 = 0} \qquad \mathbf{x - 4 = 0}$$
$$\underline{\mathbf{-1 \ \ -1}} \qquad \underline{\mathbf{-4 \ \ -4}}$$

Then solve both for 0 to find our solutions.

$$\mathbf{\frac{2x}{2} = \frac{-1}{2}} \qquad \mathbf{x = 4}$$

$$\mathbf{x = -1/2}$$

Your turn: $\mathbf{2x^2 + 7x = -4}$

[You should get x = -3 and -1/2]

Your turn: $3x^2 + 13x = 10$ (hint, make sure to solve the equation for zero before using the quadratic equation)

[You should get $x = -5$ and $2/3$]

Applications

Quadratics functions are often used to model many situations. They can be solved using the methods above.

Geometry – area/perimeter:

Let's take a look at the following problem:

> Your dorm room has a **perimeter (P)** of 46ft and an **area (A)** of $120ft^2$. What are the dimensions of your dorm?

At first this may not look like a quadratic at all! But as we work through the problem, we'll see how it relates.

$$\mathbf{P = 2L + 2w}$$

$$46 = 2L + 2w$$

$$23 = L + w$$

> We know from geometry that the formula for perimeter is **P = 2L + 2w.**
>
> Since our perimeter is 46, we just plug that number into the formula.
>
> 2 is a common factor on all sides, so we divide both sides by 2.

What else do we know….Area

$$\mathbf{A = (L)(w)}$$

$$120 = (L)(w)$$

Now, we have a system of equations:

Another quadratic application:

The height, h, of an arrow shot up in the air can be approximated by the equation $h = 128t - 16t^2$ where t is time in seconds. How long does it take for the arrow to reach 240 feet in the air?

First, we need to determine what the question is talking about. There are two variables here, t, and h. The height of the arrow depends on the time it is in the air, so we are talking about (t, h).

Next, we need to figure out what information we are given. We have the equation: $h = 128t - 16t^{2.}$ Additionally, we are told that the arrow is to reach 240 feet in the air – that is height. So we are trying to find the time that the arrow reaches 240 feet or (__, 240).

By substituting into the equation,

$$240 = 128t - 16t^2$$

We can solve using the quadratic formula. We need to solve the equation for zero before we begin the quadratic formula.

$$-16t^2 + 128t - 240 = 0$$

So, $a = -16$, $b = 128$ and $c = -240$

We will find solutions to this quadratic equation at

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-\ 128\ \pm \sqrt{128^2 - 4\ \ -16\ \ \ -240}}{2\ \ -16}$$

$$t = \frac{-128 \pm \sqrt{16384 - 15360}}{-32}$$

$$t = \frac{-128 \pm 32}{-32}$$

$$t = 3 \quad \text{or} \quad t = 5$$

So the arrow is at 240 feet in the air twice, once at 3 seconds on its way up, and again at 5 seconds on its way down.

$$\frac{(3x + 2)(x - 6)}{(x - 6)(x - 6)}$$

First we factor.

Next, find the values that make the denominator 0. In this case, the only number that makes the denominator 0 is 6, so our **domain is all real numbers except 6.**

$$\frac{(3x + 2)\cancel{(x - 6)}}{\cancel{(x - 6)}(x - 6)}$$

$$\frac{3x+2}{x-6}$$

Since $\frac{x-6}{x-6}$ equals one we can cancel leaving us with **3x + 2 over x – 6**. Doesn't that look much nicer?

Your turn: Simplify $\frac{x^2+5x+6}{x^2+8x+15}$

[You get $\frac{x+2}{x+5}$]

Multiplying and dividing rational expressions

To multiply rational expressions you multiply straight across, numerator to numerators and denominators to denominators. You may cancel any factors you find that are the same (top- to- bottom) since you are MULTIPLYING. (Don't forget to find values of the domain that make the denominator equal to zero!)

For example:

$$\frac{x^2+x-6}{2x^2-3x+1} \times \frac{x^2+3x-4}{2x^2-5x+2}$$

Always factor first!

Setting each of the factors in the denominator equal to zero we find our domain is all real numbers except for 1, 2, and ½.

$$\frac{(x+3)(x-2)}{(x-1)(2x-1)} \times \frac{(x-1)(x+4)}{(x-2)(2x-1)}$$

$$\frac{(x+3)}{(2x-1)} \times \frac{(x+4)}{(2x-1)}$$

Since **x – 1** and **x – 2** appear in both the top and bottom, we can cancel them out.

Collect what is left, and there we have it!

$$\frac{(x+3)(x+4)}{(2x-1)^2}$$

Adding/Subtracting rational expressions:

When adding/subtracting rational expressions they must have a common denominator. If the terms already have a common denominator we are free to just add them, the denominators remain the same.

$$\frac{x+2}{15}+\frac{4x-8}{15}$$

$$=\frac{x+2+4x-8}{15}$$

It is just as though we were adding: $\frac{1}{5}+\frac{3}{5}=\frac{4}{5}$, the denominator stays the same.

Then simplify;

$$=\quad\frac{5x-6}{15}$$

If the denominators are different, we must find a common denominator.

$$\frac{8}{x+6}-\frac{5}{x-6}$$

Since $\frac{8}{x+6}$ is missing a factor of (x-6) we multiply this term by $\frac{x-6}{x-6}$ (Anything divided by itself equals one and does not change the value of the equation).

We do a similar step with

$\frac{5}{x-6}$ multiplying this term by it's missing factor $\frac{x+6}{x+6}$.

$$\frac{8}{x+6}\left(\frac{x-6}{x-6}\right)-\frac{5}{x-6}\left(\frac{x+6}{x+6}\right)$$

The common denominator for this expression will be (x+6) (x-6) because these terms are NOT alike. Think of it like it we do simple fractions:

$$\frac{1}{3}+\frac{2}{5}=-$$

How did we find the common denominator then? We multiplied each term by the missing factor, just like we will now.

$$-2x^2 + 22x + 24 = 0$$

Since this is quadratic, we can either factor it or use the quadratic formula to solve. Let's see if it factors nicely…

$$-2(x^2 + 11x + 12) = 0$$

$$-2(x - 12)(x + 1) = 0$$

x = 12 and x = -1

We need to check to see if either solution makes our original equation undefined. They don't so we have two valid solutions to the equation.

Your turn: $\dfrac{2x}{x+2} - 2 = \dfrac{x-8}{x-2}$

[you get x= -4, 6]

Radical Expressions

Radical Expressions are expressions with roots of variables in them. The *square root* is a number you can *square* to get another number. For instance, the square root of 25, $\sqrt{25}$, is 5, because if you square 5, you get 25.

There is one very basic rule when dealing with roots. If the root is **even**, you **cannot have a negative number**. This means that there can be no negative numbers in square roots, fourth roots, sixth roots, etc. Why? Because if you multiply two numbers to get a negative number, one must be positive and one must be negative, right? When you square a number, you multiply the number by *itself*, so it's impossible to have one be negative and one be positive.

What about **odd roots**? If you are taking a *cube root* (third root), can you have a negative number? What would happen if you cube (-1)? (-1)(-1)(-1) = -1, so you can have a negative when dealing with odd roots.

Solving Radical Equations

When solving radical equations we raise both sides of the equation to the power of the radical, but before we do that we need to get the radical on one side of the equation by itself. It is also important that we **check our solution** to make sure that it exists in our domain (that is, make certain our solution does not make our radical undefined).

For example:

$$\sqrt{x}-6=2$$

$$\sqrt{x}=8$$

$$\sqrt{x}^{\;2}=8^2$$

$$x=64$$

If we square both sides BEFORE we isolated the radical we would need to foil the product…It is much easier if we move everything first ☺

Check: $\sqrt{64}-6=2$

More practice:

$$\sqrt[3]{2x+1}-2=3$$

$$\sqrt[3]{2x+1}=5$$

$$2x+1^{\frac{1}{3}}=5$$

$$\left[\;2x+1^{\frac{1}{3}}\right]^3=5^3$$

$$2x+1=5^3$$

$$2x = 124$$

$$x = 62$$

Let's get the radical on one side first. Then we will cube both sides of the equation and solve for x.

Checking our solution to make sure that it is in our domain…

Check: $\sqrt[3]{2\;62+1}-2=3$

Your turn: $\sqrt{3y+1}+6=10$

[you get y=5]