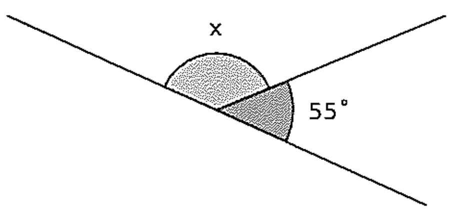
Rules of angles (7–9)

Contents

1 basic rules of angles

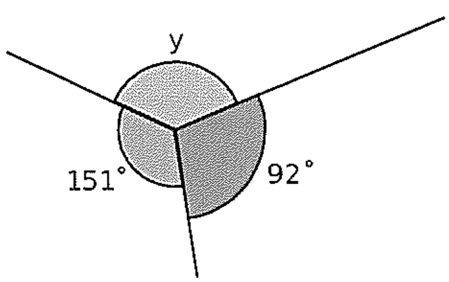
There are various *Rules of angles* that you should know. These can be used in any geometrical diagram to work out missing angles without the diagram having to be drawn to scale. We do not need a protractor since the rule will give us the exact answer. The basic rules you should know are:

Angles on a straight line add to $180°$



$$x + 55 = 180 \qquad \textit{Angles on a straight line}$$
$$x = 125°$$

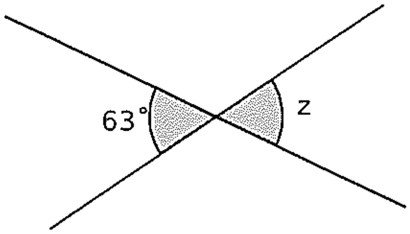
Angles at a point add to $360°$



$$y + 92 + 151 = 360 \qquad \textit{Angles at a point}$$
$$y + 243 = 360$$
$$y = 117°$$

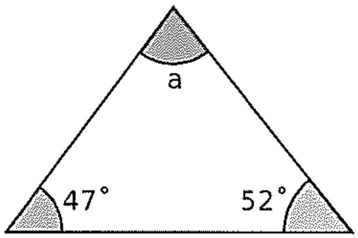
Vertically opposite angles are equal

Note: this is not like angles at a point since here we are dealing with where two straight lines intersect, like a pair of scissors:



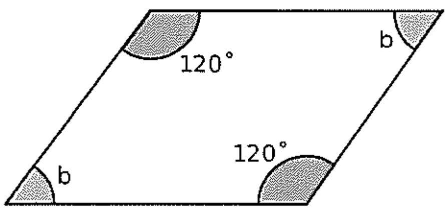
$$z = 63^\circ \qquad \textit{Vertically opposite angles}$$

Angles in a triangle add to 180°



$$\begin{aligned} a + 47 + 52 &= 180 \qquad \textit{Angles in a triangle} \\ a + 99 &= 180 \\ a &= 81^\circ \end{aligned}$$

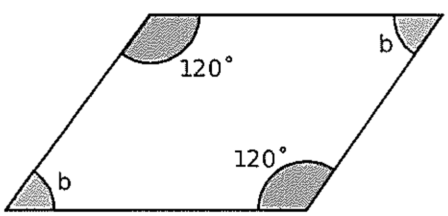
Angles in a quadrilateral add to 360°



$$\begin{aligned} b + 120 + b + 120 &= 360 \quad \textit{Angles in a quadrilateral} \\ 2b + 240 &= 360 \\ 2b &= 120 \\ b &= 30^\circ \end{aligned}$$

Notice how, in each case, we set out our working clearly using a logical algebraic layout and we always give the reason for a particular angle.

Example. Find x and y in the following diagram:



To find x:

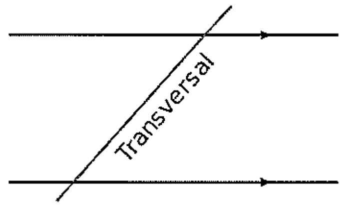
$$\begin{aligned} x + 75 &= 180 \qquad \textit{Angles on a straight line} \\ x &= 105^\circ \end{aligned}$$

To find y:

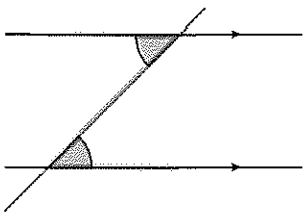
$$y = 85^\circ \qquad \textit{Vertically opposite angles}$$

2 Angles in parallel lines (7–9)

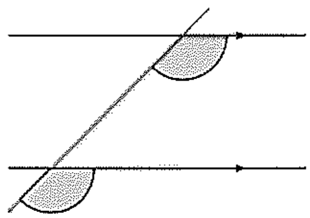
When a line passes through a pair of parallel lines, this line is called a transversal:



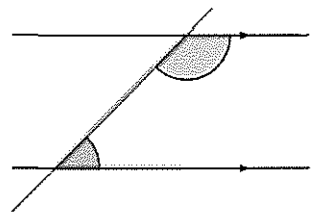
A transversal creates three letters of the alphabet which hide 3 new rules of angles:



Alternate angles are equal *(Z-angles)*

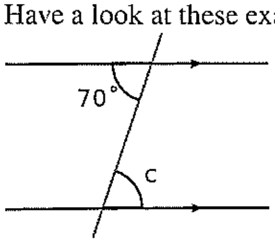


Corresponding angles are equal *(F-angles)*

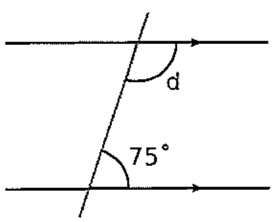


Interior angles add to 180° *(C-angles)*

Have a look at these examples:

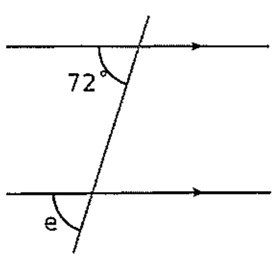


$c = 70°$ *Alternate angles*



$d + 75 = 180$ *Interior angles*

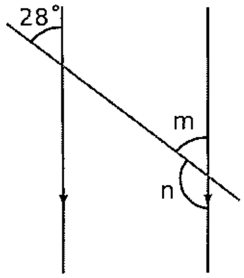
$d = 105°$



$e = 72°$ *Corresponding angles*

$d = 105°$

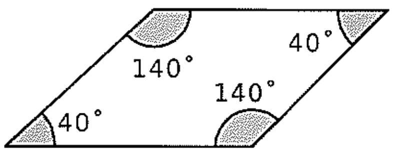
Note that the "F" is back to front!



$$m = 28^\circ \qquad \textit{Corresponding angles}$$
$$m + n = 180^\circ \qquad \textit{Angles on a straight line}$$
$$n = 152^\circ$$

Angles in quadrilaterals

We have already seen that the angles in any quadrilateral add up to 360°. There is an interesting special case that allows us to use what we have just learned about angles in parallel lines:



In a parallelogram, angles next to each other make a "C" shape (interior angles). This means that they add up to 180°. Therefore,

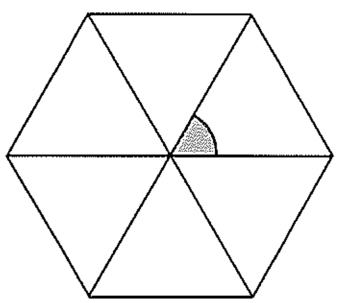
In a parallelogram, opposite angles are equal.

3 Angles in polygons (year 9)

- A *polygon* is a shape with straight sides.
- A *regular polygon* has all sides and all angles equal.

We may need to find several angles in polygons.

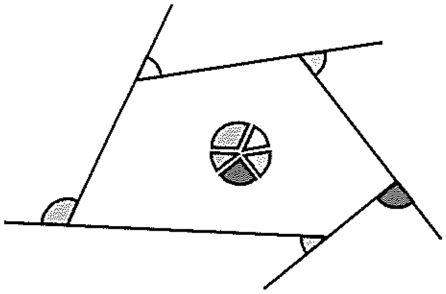
3.1 The central angle in a regular polygon



The angles sit around a circle and so add to 360°. Each angle is $360 \div n$, where n is the number of sides of the polygon.
E.g. here we have a hexagon:

$$\text{Each angle is } 360 \div 6 = 60^\circ$$

3.2 The exterior angle of any polygon



In any polygon, the exterior angles are found where the extension of a side meets the next side, as the diagram shows. Since these extensions all form a "windmill" effect, their total turn is equivalent to a full circle.

Sum of exterior angles $= 360°$

Example. What is the exterior angle of a regular pentagon?

Each angle is equal as the pentagon is regular. Therefore,

$$\begin{aligned} \text{Each angle} &= 360 \div 5 \\ &= 72° \end{aligned}$$

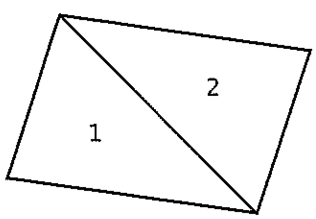
3.3 The interior angle of any polygon

We know that:

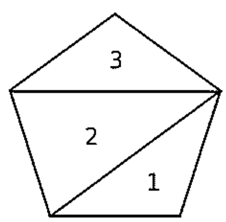
- in a triangle, interior angles add to 180°;
- in a quadrilateral, interior angles add to 360°.

If we follow the pattern, we notice that the total goes up by 180° each time.

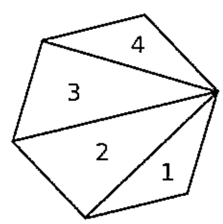
But why is this? If we take one vertex of any polygon and join it to all of the others, we create triangles:



Quadrilateral
2 triangles: $2 \times 180 = 360°$



Pentagon
2 triangles: $3 \times 180 = 540°$



Hexagon
2 triangles: $4 \times 180 = 720°$

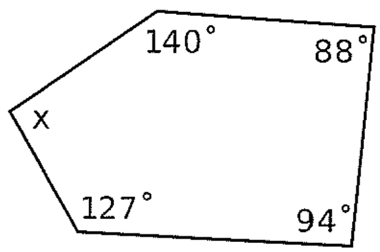
Notice also that the number of triangles needed is always two less than the number of sides in the polygon. So in general:

$$\begin{bmatrix} \text{Sum of} \\ \text{interior angles} \end{bmatrix} = 180(n-2), \quad \text{where } n \text{ is the number of sides}$$

Moreover, if the polygon is regular, we can divide the sum by n to obtain the size of each interior angle. The following table sums these up for a few polygons:

Number of sides	n	3	4	5	6	7	8	9	10
Number of triangles	$n-2$	1	2	3	4	5	6	7	8
Sum of angles	$180(n-2)$	180	360	540	720	900	1080	1260	1440
Each angle if regular	$\frac{180(n-2)}{n}$	60	90	108	120	128.57	135	140	144

Example. What is the missing angle below?



In a pentagon, the sum of the interior angles is 540°.

$$\begin{aligned} x + 135 + 130 + 75 + 120 &= 540 \\ x + 460 &= 540 \\ x &= 80^\circ \end{aligned}$$

Example. What is the size of any interior angle in a regular dodecagon? (NB A dodecagon has 12 sides)

A 12 sided shape can be divided into 10 triangles.

$$\begin{aligned} \text{Sum of interior angles} &= 10 \times 180^\circ \\ &= 1800^\circ \end{aligned}$$

Therefore

$$\begin{aligned} \text{Each interior angle} &= 1800 \div 12 \\ &= 150^\circ \end{aligned}$$

Using Angle Facts

Lines	Triangles & Quadrilaterals	Parallel lines
Vertically Opposite Angles are equal	Angles in a triangle add up to 180°	Alternate angles are equal
Angles on a straight line add to 180°	Base angles of an isosceles triangle are equal	Corresponding angles are equal
Angles at a point add up to 360°	Angles in an equilateral triangle are equal	Interior angles add to 180°
	Angles in a quadrilateral add up to 360°	

One or two of the reasons above must be quoted in **all** examinations involving angle problems when the problem requires an explanation.

For the following problems, find the missing angles and give a reason to go with each angle (choose it from the list above). Please note – calculations ARE NOT reasons.

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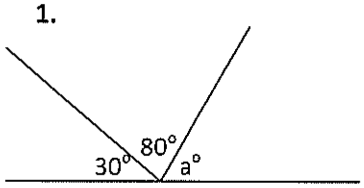

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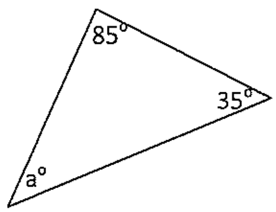

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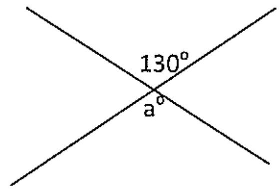

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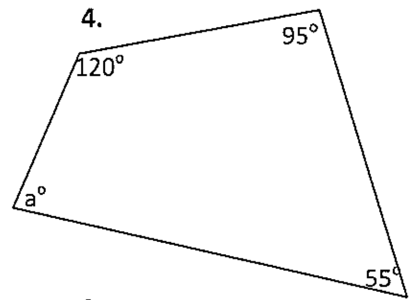

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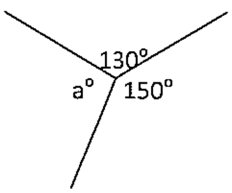

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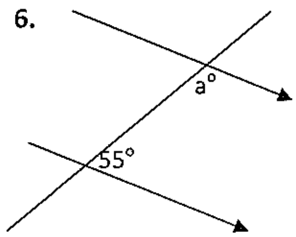

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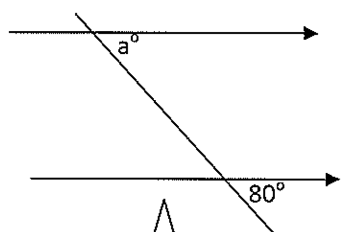

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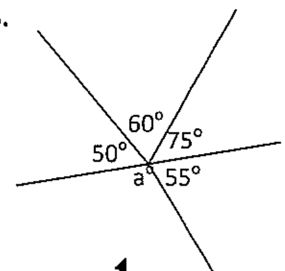

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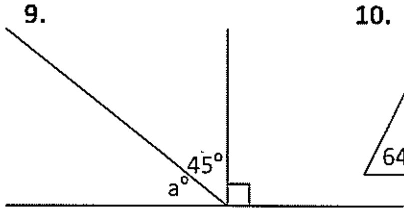

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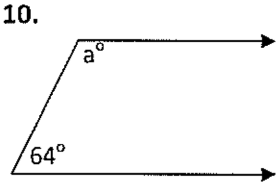

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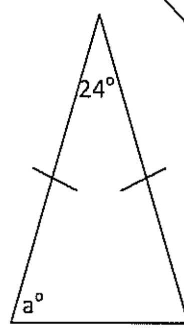

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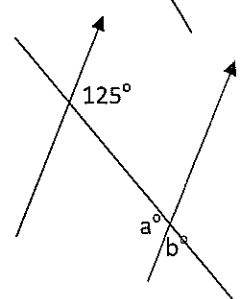

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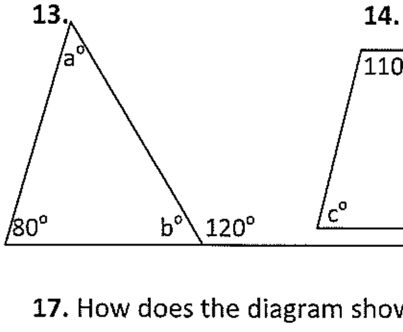

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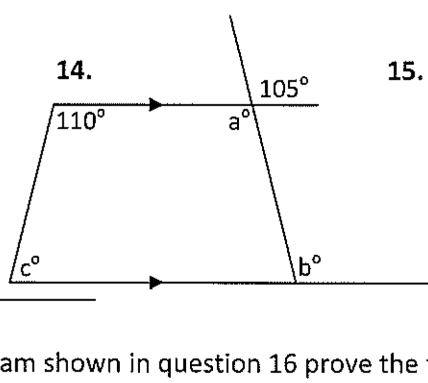

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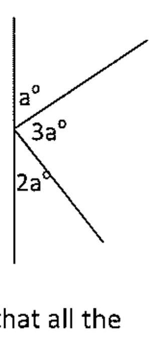

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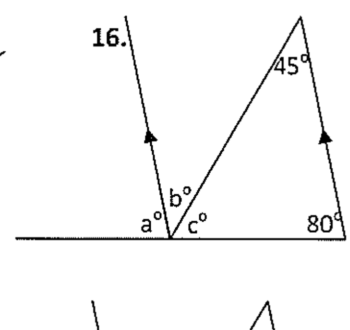
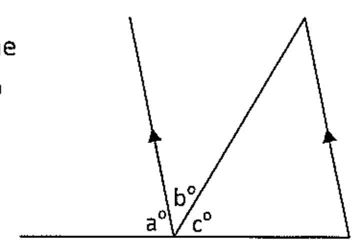
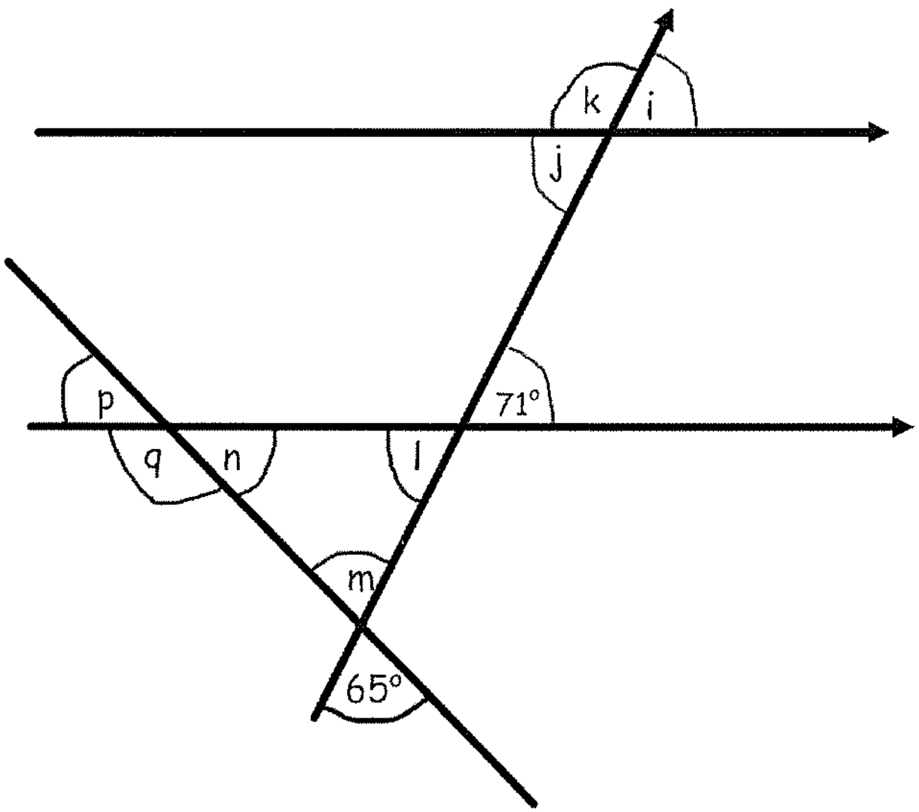


17. How does the diagram shown in question 16 prove the fact that all the angles in a triangle add up to 180°. Copy the following diagram and try to explain mathematically why.



EXTENSION TASK: Find the missing angles below, in any order.
Circle if the angle is alternate, corresponding, or vertically opposite to which other angle.



i = ... alternate/corresponding/vertically opposite to ....

j = ... alternate/corresponding/vertically opposite to ....

k = ... alternate/corresponding/vertically opposite to ....

l = ... alternate/corresponding/vertically opposite to ....

m = ... alternate/corresponding/vertically opposite to ....

n = ... alternate/corresponding/vertically opposite to ....

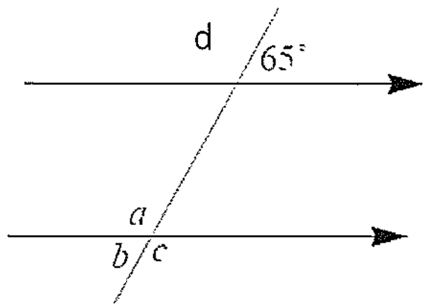
p = ... alternate/corresponding/vertically opposite to ....

q = ... alternate/corresponding/vertically opposite to ....

Angle Rules and Parallel Lines: True or False?

You are given some statements about Diagram 1 and you have to decide which are true and which are false. When you have done this for diagrams 1, 3, 5, 7 and 9 you should then make up some statements of your own for the other diagrams

1)



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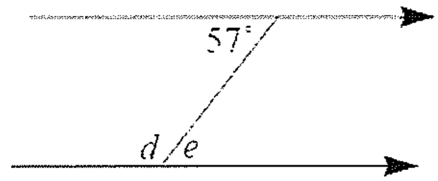
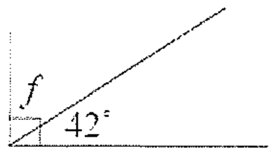


Diagram 1 Angle b = 115^0	Diagram 1 Angles b and c are equal
Diagram 1 Angles a and d are equal because they are alternate angles	Diagram 1 Angles a + b = 180^0 because they form a straight line
Diagram 1 Angle a = 115^0	Diagram 1 Angles a and d are equal because they are corresponding angles
Diagram 1 Angles d + 65^0 = 180^0 because they form a straight line	Diagram 1 Angle c = 65^0 because c and 65^0 are corresponding angles
Diagram 1 Angles b + c = 180^0 because they form a straight line	Diagram 1 Angle d = 115^0
Diagram 1 D = 180^0 - 65	Diagram 1 Angle a = c because they are opposite angles

3)



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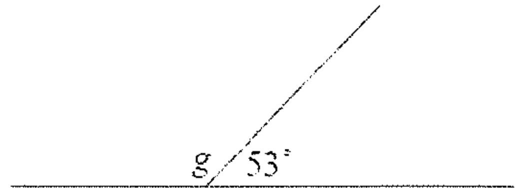
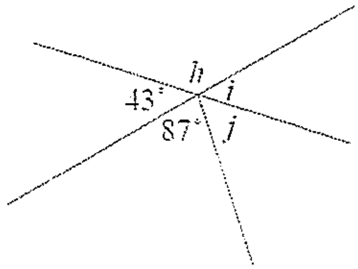


Diagram 3 $180^0 - 42^0 = f$	Diagram 3 Angle f = 58^0
Diagram 3 There is a right angle in the diagram	Diagram 3 A right angle is 90^0
Diagram 3 $f = 42^0$	Diagram 3 Angles $f + 42^0 = 180^0$ because they form a straight line
Diagram 3 Angles $f + 42^0 = 90^0$ because they make a right angle	Diagram 3 Angle f = 48^0

5)



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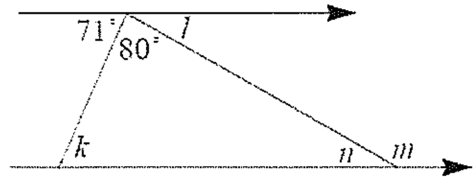
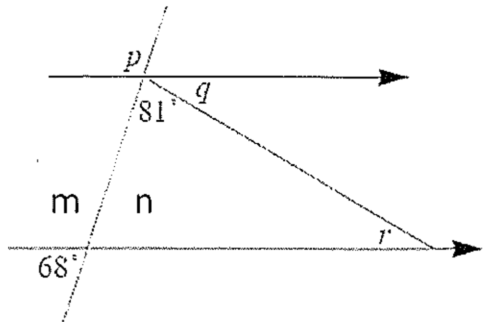


Diagram 5 Angle h = 87^0 because it is opposite the angle 87^0	Diagram 5 j + 87^0 + 43^0 = 180^0 because they form a straight line
Diagram 5 Angle i = 43^0 because they are opposite angles	Diagram 5 Angles j + i = 90^0 because they make a right angle
Diagram 5 Angle h = 137^0	Diagram 5 Angle h is the same as j + 87^0 because they are opposite angles
Diagram 5 Angles h + i = 180^0 because they form a straight line	Diagram 5 Angle j = 60^0
Diagram 5 Angle h = 180^0 - 43^0	Diagram 5 All of the angles add up to 360^0

7)



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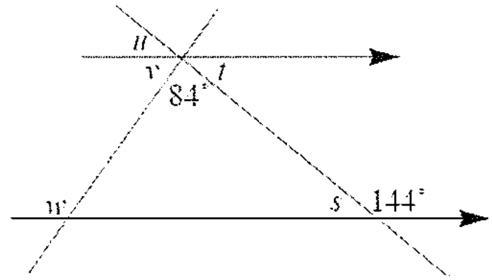
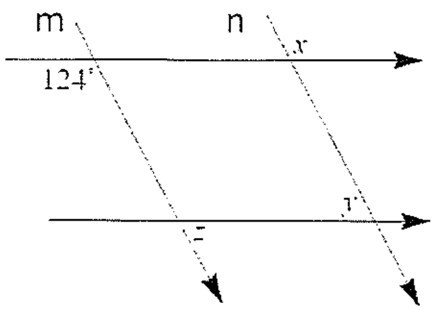


Diagram 7 Angle q + $81^0 = 90^0$	Diagram 7 r = 31^0
Diagram 7 81^0 + n + r = 180^0	Diagram 7 Angle p = 81^0 + q because they are opposite
Diagram 7 Angles r and q are equal because they are alternate angles	Diagram 7 n = 81^0 because there is an isosceles triangle
Diagram 7 m = 112^0	Diagram 7 Angle m + 68^0 = 180^0 as they form a straight line
Diagram 7 Angle p = 68^0 because they are corresponding angles	Diagram 7 q = 68^0
Diagram 7 Angle m = q + 81^0 as they form alternate angles	Diagram 7 Angle n = 68^0 because it is opposite the 68^0

9)



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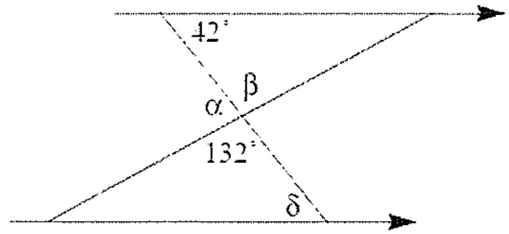

Diagram 9 Angle z and y are called alternate angles	Diagram 9 Angle m = 66^0
Diagram 9 Angle z = 124^0	Diagram 9 Angle x = Angle y
Diagram 9 The angle opposite the 124^0 corresponds to angle x	Diagram 9 Angles x and y are called alternate angles
Diagram 9 m + 124^0 = 180^0 as they form a straight line	Diagram 9 Angle x is opposite the 124^0
Diagram 9 Angles m and n are corresponding angles	Diagram 9 Angle n = 56^0
Diagram 9 There are only 2 different sized angles in this diagram	Diagram 9 Angles n and y are corresponding angles